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ELECTRO-PHYSICS LABORATORIES

EXTERNAL TECHNICAL MEMORANDUM NO. E15

NOMOGRAMS FOR OBTAINING THE ARRIVAL ANGLE AND BEARING OF RADIO WAVES FROM MEASUREMENTS OF PHASE DIFFERENCE BETWEEN THREE STATIONS ARRANGED IN AN EQUILATERAL TRIANGLE

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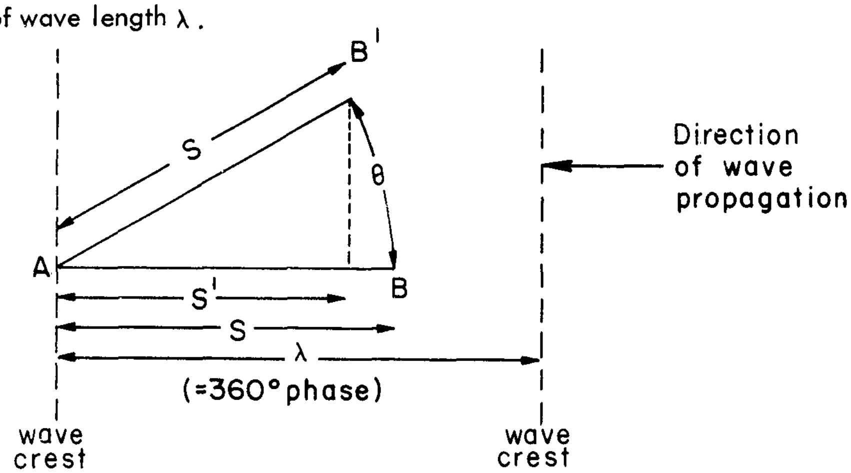
1. INTRODUCTION

The nomographic procedures described here are devised for facilitating computation of values for wave arrival angles obtained from a 3-station receiver system. The original effort was conducted to simplify the resolution of the angle-of-arrival vector from component measurements made on a wideband "3-port" receiving system at the Electro-Physics Laboratories' field site (Ref. 1). The particular 3-port configuration utilized three wideband, vertically polarized, monopole antennas placed at the apices of an equilateral triangle.

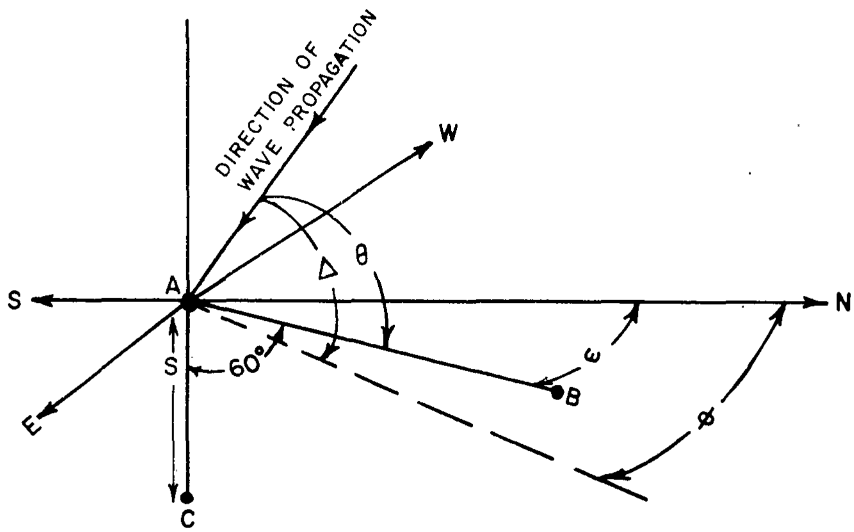
2. CONSTRUCTION AND USE OF NOMOGRAMS

Suppose there are two observing stations, A and B, at distance S from each other, which lie along the direction of propagation of a train of plane radio waves, of wave length λ.



The difference in phase of the wave train observed at A from that observed at B will be equal to $\frac{360°S}{\lambda}$. If the distance, S, between A and B does not lie along the direction of propagation of the train of waves, the observed phase difference will be less if S and λ are kept constant. Thus, if the observing station B is moved to B' at the same distance, S, from A, but at an angle θ with the direction of wave propagation, the observed phase difference will be $\frac{360° S'}{\lambda}$, where $S' = S \cos \theta$.

Let us now consider such a problem in three dimensions, where A and B are two ground observing stations in the horizontal plane, and the direction of propagation of the wave train is from above, at an arrival angle of Δ with the horizontal plane, and at a bearing angle, ϕ, with some reference direction (customarily taken as true north). Let station B lie at a distance, S, from station A, as before, at a bearing, ω, from the reference direction (true north). The phase difference observed between A and B will be $\frac{360° S\cos\theta}{\lambda}$, as before, where θ is the angle between the direction AB and the direction of wave propagation, and $\cos\theta = \cos\Delta\cos(\phi-\omega)$.



For present experimental work three stations are employed, A, B, and C, situated at the corners of an equilateral triangle, the length of the sides of the triangle being S. The difference in phase observed between stations A and B, δ_1, and that observed between stations A and C, δ_2, is

$$\delta_1 = \frac{360° S \cos\Delta \cos(\phi-\omega)}{\lambda}$$

$$\delta_2 = \frac{360° S \cos\Delta \cos(\phi-\omega-60°)}{\lambda}$$

To determine the arrival angle, Δ, and the bearing, ϕ, of the incoming radio wave, first subtract δ from δ_2, thus obtaining D.

$$\delta_2 - \delta_1 = D = \frac{360^\circ S}{\lambda} \cdot \cos \Delta \cdot \left[\cos(\phi - 60^\circ - \omega) - \cos(\phi - \omega)\right]$$

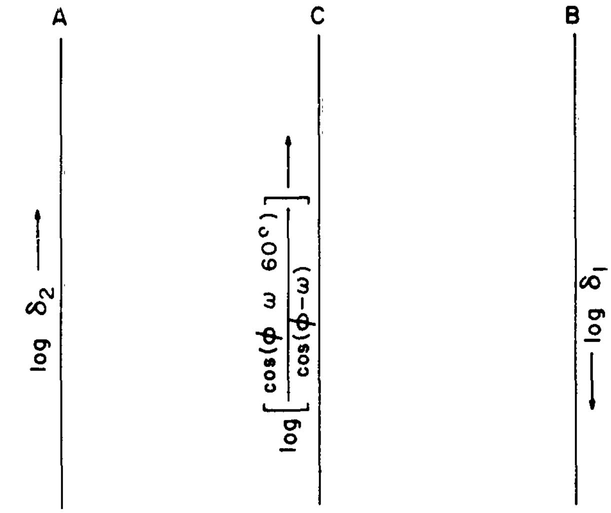
Also

$$\delta_2 / \delta_1 = \frac{(\cos\ \phi - \omega - 60^\circ)}{\cos(\phi - \omega)}$$

These two equations may be solved nomographically to obtain Δ and ϕ.

2.1. NOMOGRAM I

$$\delta_1 / \delta_2 = \frac{\cos(\phi - \omega - 60^\circ)}{\cos(\phi - \omega)}$$

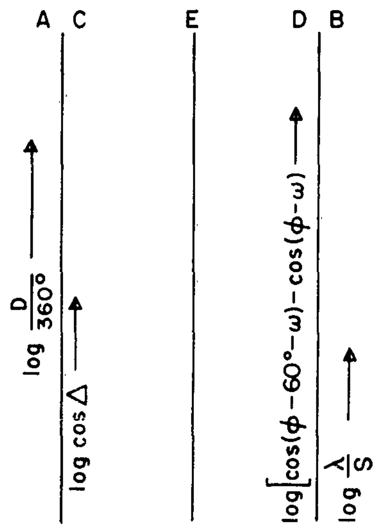


To construct: Scales A and B are identical logarithmic scales, progressing in opposite directions, each two log-cycles in length. Scale C is midway between them and will be 4 log-cycles in length (i.e., the size of a log-cycle on it will be half as great). Scale C will have distances on it proportional to log $\left[\frac{\cos(\phi - \omega - 60^\circ)}{\cos(\phi - \omega)}\right]$, but they should be labelled in corresponding values of ϕ. To know where to start this scale on the central line, find the value of ϕ, algebraically, corresponding to two values of scales A and B (the bottom ones, for convenience). (It is also a good idea to solve, algebraically, for the value of ϕ at some other, distant, pair of values of δ_1 and δ_2, as a check on the construction of the chart.)

To use: Align values of δ_1 and δ_2. ϕ may be read directly from the central scale.

2.2. NOMOGRAM II

$$\frac{D}{360^\circ} \cdot \frac{\lambda}{S} = \cos\Delta \cdot \left[\cos(\phi - 60^\circ - \omega) - \cos(\phi - \omega)\right]$$



To construct: Nomogram II is a composite of the same types as Nomogram I. Since multiplication (rather than division) is involved, scales A and B, and scales C and D progress in the same (not opposite) directions. The plan is $A \cdot B = E$;

C' D=E. In constructing the nomogram at least one value (preferably also another for check purposes) should be put on scale E. After it is assured that the diagram is correct, these values may be erased, leaving E simply as a bare, unscaled line. To use: Align the value of D on scale A with that of λ (or the equivalent frequency) on scale B, and mark the intersection (O) of this line with the unmarked scale E. Align the value of ϕ, previously determined from nomogram I, on scale D, with O on E, and project to scale C to determine Δ.

The accompanying nomograms are made for the 3-port configuration illustrated on Nomogram I, where the distance, S, between ports is 88.7 feet.

3. ADDENDUM

The preceding instructions for construction have been given for convenience and simplicity. They are not the only nomographic representations possible, however, for the equations represented. These equations are of the type $f_1 f_2 f_3 = 1$, where subscripts 1, 2, and 3 refer to the three variables concerned, f_1 indicating, for example, a function of variable 1. Nomogram II is a composite of two equations of this type.

One way this type of equation may be solved nomographically is by using an alignment chart having two parallel scales and a slant scale intersecting the parallel scales at their zero values of the functions represented on them. This nomographic form has the disadvantage of having a scale function on the slant scale for which a range of values becomes extremely compressed on one end, so that solutions within this region are of poor accuracy.

A more practical way of solving such equations, for our present purpose, is by stating the equation in logarithmic form. Stated thus, the equation has the canonical form $f_1 + f_2 + f_3 = 0$, which may be represented by an alignment chart in three parallel scales.

If the equation is given in the form $l_1 f_1 + l_2 f_2 + l_3 f_3 = 0$, where l_1, l_2, and l_3 are nomographic scale factors, the equation may be represented in determinantal form as

$$\begin{vmatrix} 0 & l_1 f_1 & 1 \\ d & l_2 f_2 & 1 \\ \dfrac{d\, l_1}{l_1 + l_2} & -l_3 f_3 & 1 \end{vmatrix} = 0 \text{ , where } l_3 = \frac{l_1 \; l_2}{l_1 + l_2} ,$$

and d is the width, arbitrarily chosen, of the alignment chart. The scale factors for two of the nomographic scales may be arbitrarily chosen, but not that for the third.

Expressing the equation in the preceding determinantal form has the advantage of representing the x and y values of the function scale for each variable, on a Cartesian plot, by the first two column terms for the row pertaining to each variable. This circumstance results from the fact that the equation for a straight line may be given in similar determinantal form as

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{x_1 - x_2}{y_1 - y_2} - \frac{x_2 - x_3}{y_2 - y_3} = 0$$

Solution of the equation represented by any alignment chart must correspond to the colinearity of points on the function scales of the variables concerned. The determinantal form given for the equation concerned, thus, assures one that such colinearity is possible for the alignment-chart form indicated.

4. REFERENCES

1. Green, J.A., "Theoretical Patterns for Null Mode Operations of 16-Element Linear Array," ITM 121, Electro-Physics Laboratories, Hyattsville, Maryland, 23 April 1963.

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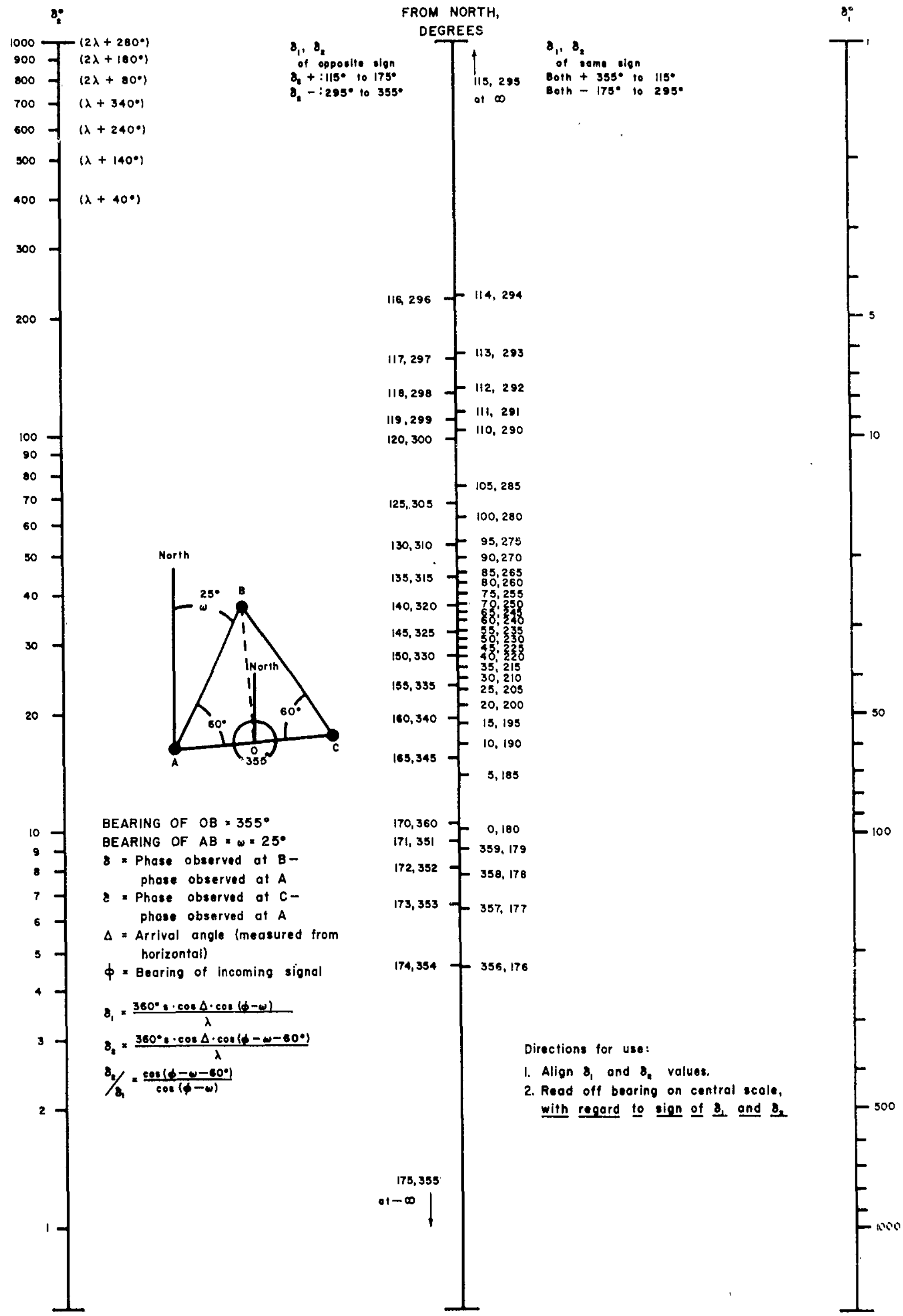
NOMOGRAM I



BEARING OF OB = 355°

BEARING OF AB = ω = 25°

δ_1 = Phase observed at B − phase observed at A

δ_2 = Phase observed at C − phase observed at A

Δ = Arrival angle (measured from horizontal)

φ = Bearing of incoming signal

$$\delta_1 = \frac{360^\circ s \cdot \cos\Delta \cdot \cos(\phi-\omega)}{\lambda}$$

$$\delta_2 = \frac{360^\circ s \cdot \cos\Delta \cdot \cos(\phi-\omega-60^\circ)}{\lambda}$$

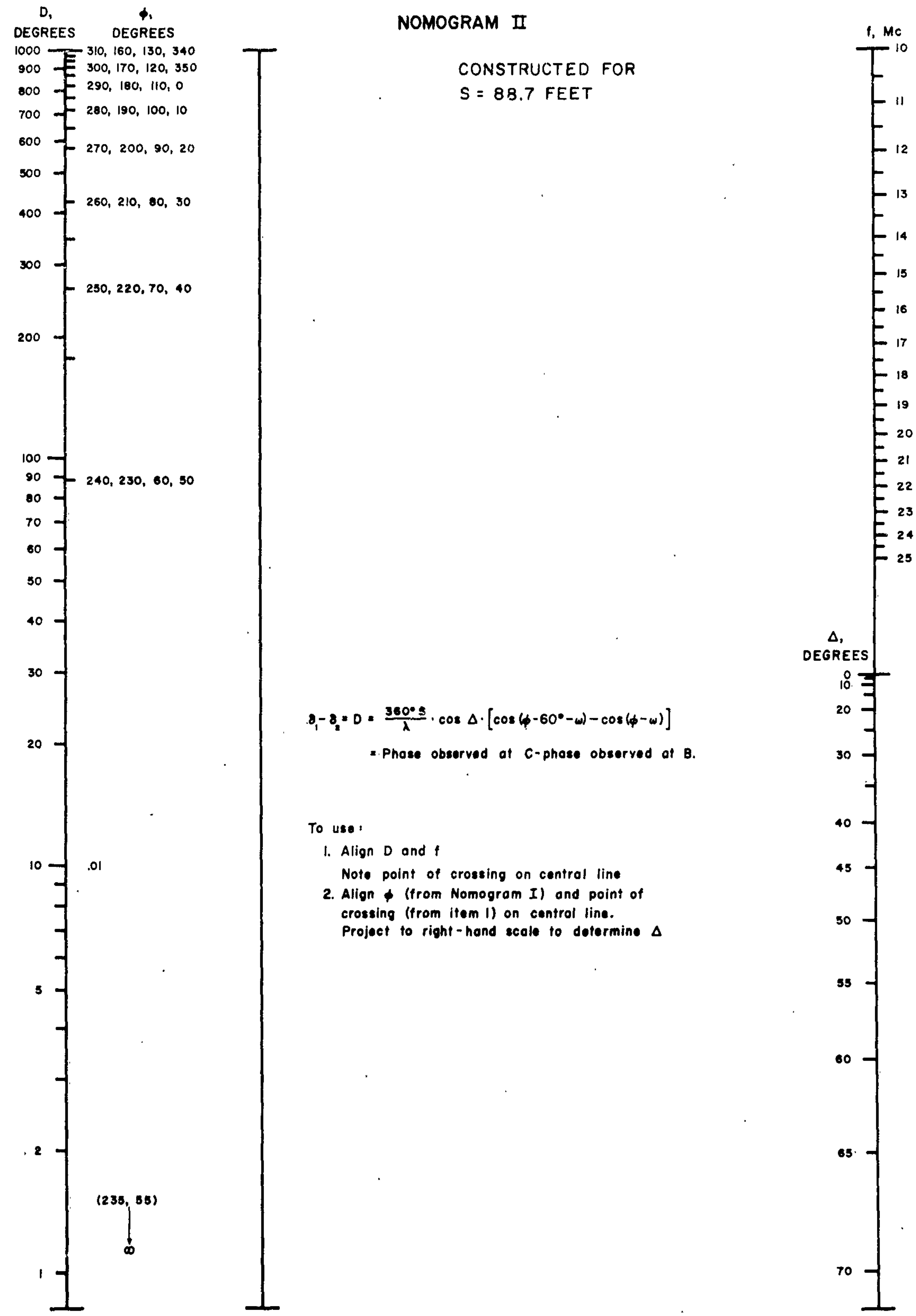
$$\delta_2/\delta_1 = \frac{\cos(\phi-\omega-60^\circ)}{\cos(\phi-\omega)}$$

Directions for use:

1. Align δ_1 and δ_2 values.
2. Read off bearing on central scale, <u>with regard to sign of δ_1 and δ_2</u>.

NOMOGRAM II

CONSTRUCTED FOR
S = 88.7 FEET



$$\delta_1 - \delta_2 = D = \frac{360° S}{\lambda} \cdot \cos \Delta \cdot \left[\cos(\phi - 60° - \omega) - \cos(\phi - \omega) \right]$$

= Phase observed at C - phase observed at B.

To use:

1. Align D and f
   Note point of crossing on central line
2. Align ϕ (from Nomogram I) and point of crossing (from item 1) on central line.
   Project to right-hand scale to determine Δ